School of Mathematical Sciences
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Examiner: Dr A. Treglown

MTH5105 Differential and Integral Analysis
MID-TERM TEST

Date: 22 Feb 2013 *Time:* 15:10–15:50

Complete the following information:

Name	
Student Number (9 digit code)	

The test has THREE questions. You should attempt ALL questions. Write your calculations and answers in the space provided. Cross out any work you do not wish to be marked.

Question	Marks
1	
2	
3	
Total Marks	

Nothing on this page will be marked!

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Question 1.

(a) State the formula for the Taylor polynomial $T_{n,a}$ of degree n of a function f at the point a.

[*10 marks*]

Let $f : [0, \infty) \to \mathbb{R}$ be defined by $f(x) = 1/\sqrt{1+2x}$.

(b) Determine the Taylor polynomials $T_{2,0}$ and $T_{3,0}$ of degree 2 and 3, respectively, for f at $a = 0$. [*15 marks*]

(c) Using the Lagrange form of the remainder term, or otherwise, show that

$$T_{3,0}(x) < f(x) < T_{2,0}(x) \quad \text{for all } x > 0 \ .$$

[*10 marks*]

Answer 1.

Answer 1. (*Continue*)

Question 2.

(a) Give the definition of $f : \mathcal{D} \to \mathbb{R}$ being differentiable at a point $a \in \mathcal{D}$. [*10 marks*]

(b) Define $f : \mathbb{R} \to \mathbb{R}$ by $f(x) = \begin{cases} 2x^3 \cos\left(\dfrac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$.

Prove that $f'(0) = 0$. [*15 marks*]

(c) Define $g : \mathbb{R} \to \mathbb{R}$ by $g(x) = \begin{cases} 2x \cos\left(\dfrac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$.

Is g differentiable at 0? Briefly justify your answer. [*10 marks*]

Answer 2.

Answer 2. (*Continue*)

Question 3.

(a) State the Mean Value Theorem. [*15 marks*]

(b) Suppose that $0 < a < b$. By applying the Mean Value Theorem to the logarithm function show that
$$1 - \frac{a}{b} < \log\left(\frac{b}{a}\right) < \frac{b}{a} - 1.$$
You may assume standard properties of the logarithm function. [*15 marks*]

Answer 3.

Answer 3. (*Continue*)