MTH5105 Differential and Integral Analysis 2012-2013

Exercises 1

There are two sections. Answers to questions in Section 1 will be used for feedback. Questions in Section 2 are voluntary but highly recommended.

1 Exercises for Feedback

1) Determine at which points the following two functions are differentiable, evaluating the derivative wherever it exists.

   (a) $f : \mathbb{R} \to \mathbb{R}$, $x \mapsto (x+1)|x|$,

   (a) $g : \mathbb{R} \to \mathbb{R}$, $x \mapsto (x-1)|x-1|$.

2 Extra Exercises

2) Prove that the function $f : \mathbb{R} \to \mathbb{R}$ given by

$$f(x) = \begin{cases} x^2 \sin(1/x^2) & x \neq 0 \\ 0 & x = 0 \end{cases}$$

   is differentiable at zero and find $f'(0)$.

3) Let $f : [-1, 1] \to \mathbb{R}$ be continuous on $[-1, 1]$, differentiable at zero and $f(0) = 0$. Show that the function

$$g(x) = \begin{cases} f(x)/x & x \neq 0 \\ f'(0) & x = 0 \end{cases}$$

   is continuous at zero.

   Is g continuous for $x \neq 0$?

   Deduce that there is some number M such that

$$f(x)/x \leq M \quad \text{for all} \quad x \in [-1, 1] \setminus \{0\} \; .$$

4) Give an example of a function that is differentiable on (a, b) but that cannot be made differentiable on $[a, b]$ by any definition of $f(a)$ or $f(b)$. Can you give an example where f is bounded?

The deadline is 11am on Tuesday 22nd January. Please hand in your coursework to the orange coursework box on the second floor. Coursework will be returned during the exercise class immediately following the deadline.