MTH5105 Differential and Integral Analysis 2012-2013

Exercises 4

There are two sections. Questions in Section 1 will be used for feedback. Questions in Section 2 are voluntary but highly recommended. In particular, Question 2 formed part of a question on a previous year's **mid-term test**.

1 Exercises for Feedback

1) Let the function $f\colon (0,\pi) \to \mathbb{R}$ be given by $x \mapsto \cos(x)$. What is $f((0,\pi))$?

   Show that f is invertible and that the inverse $g = f^{-1}$ is differentiable. Find a formula for the derivative g'.

   Compute the Taylor polynomial $T_{1,0}$ for g (recall that $T_{1,0}$ denotes the degree-one Taylor polynomial at the point 0). What is the remainder term in the Lagrange form?

   Hence show that for $x \in [0, 1/2]$,

$$|g(x) - \pi/2 + x| \leq \sqrt{3}/18 \approx 0.096 .$$

2 Extra Exercises

2) Let $f(x) = \exp(1 - \exp(x)) = e^{1-e^x}$.

   (a) Determine the Taylor polynomials $T_{1,0}$, $T_{2,0}$, and $T_{3,0}$, of degrees 1, 2, and 3, respectively, for f at the point $a = 0$.

   (b) Prove that $f(x) > 1 - x$ for all $x > 0$.

3) In lectures we have shown that the number e can be expressed as

$$e = \exp(1) = \sum_{k=0}^{\infty} \frac{1}{k!} .$$

   Show that the remainder term r_n in

$$n!e = n! \sum_{k=0}^{n} \frac{1}{k!} + r_n .$$

   cannot be an integer. Hence deduce that e is irrational.

The deadline is 11am on Tuesday 12th February. Please hand in your coursework to the orange coursework box on the second floor. Coursework will be returned during the exercise class immediately following the deadline.