

# MTH5103 Complex Variables 2012-2013

## Coursework 1

Please put your solution to the *starred feedback exercise* in the red Complex Variables box in the basement by 3pm Friday 18 January. Remember to put your name (surname underlined) and student number on your solution and to staple the pages.

**Exercise 1.** Let  $z_1 = 1 + 3i$  and  $z_2 = 2 - 2i$ . Express the following complex numbers in standard form,  $x + iy$ :

$$(a) \quad z_1 z_2 \quad (b) \quad \frac{1}{z_1} \quad (c) \quad \frac{z_2}{z_1} \quad (d) \quad \frac{1}{z_1} + \frac{1}{z_2}$$

Compute the moduli:

$$(e) \quad |z_1| \quad (f) \quad |z_1 z_2| \quad (g) \quad \left| \frac{z_1 z_2}{z_1 + z_2} \right|$$

**Exercise 2.** Express the following complex numbers in polar form:

$$(a) \quad -2 \quad (b) \quad i \quad (c) \quad 1 - i \quad (d) \quad 1 + \sqrt{3}i \quad (e) \quad 1/(1 + i)^2$$

**Exercise\* 3.** Solve for the roots of the equation

$$\left( z - \frac{i}{2} \right)^3 + i = 0.$$

Express all the roots in standard form,  $x + iy$ , and draw a diagram showing their location in the complex plane.

**Exercise 4.** Describe graphically the sets of points in the complex plane defined by the following equations and inequalities:

(a)

$$\frac{3}{\bar{z}} = z$$

(b)

$$\operatorname{Im}(z^3) > 0$$

(c)

$$|z| = \operatorname{Im}(z) + \frac{1}{2}$$

Notation:  $\operatorname{Re}$  and  $\operatorname{Im}$  denote the real and imaginary parts of  $z$ , respectively.