MTH5103 Complex Variables 2012-2013

Coursework 2

Please put your solution to the *starred feedback exercise* in the red Complex Variables box in the basement by 3pm Friday 25 January. Remember to put your name (surname underlined) and student number on your solution and to staple the pages.

Exercise 1. Using Euler's formula, express in terms of $\sin\alpha$ and $\cos\alpha$ the following:

(a) $\sin 2\alpha \quad \text{and} \quad \cos 2\alpha\,;$

(b) $\sin 3\alpha \quad \text{and} \quad \cos 3\alpha\,.$

Exercise 2. (a) Find all complex solutions of the equation $e^{2z} = -1$, by writing both sides in polar form.

Using the results from (a), determine all complex solutions of

$$(b) \quad \cosh z = 0\,, \qquad (c) \quad \cos z = 0\,.$$

Exercise 3. Consider a Möbius transformation

$$z \mapsto w = \frac{az+b}{cz+d}$$

Determine the constants a, b, c and d (all $\in \mathbb{C}$), such that the transformation maps $0 \to i$, $-i \to 1$, and $-1 \to 0$.

Exercise* 4. Consider the transformation

$$z \mapsto w = z^2 + z - 3\,,$$

where as usual $z = x + iy$ and $w = u + iv$.

(a) Under the transformation, what points in the z-plane are mapped to $v = c$ in the w-plane (for c a non-zero real constant)?

(b) Find the image of the line $Re(z) = k$ (for k a real constant not equal to $-1/2$) under the transformation.

(c) Sketch your results from (a) and (b) for $c = 1$ and $k = 0$, respectively.

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