MTH5103 Complex Variables 2012-2013

Coursework 3

Please put your solution to the *starred feedback exercise* in the red Complex Variables box in the basement by 3pm Friday 1 February. Remember to put your name (surname underlined) and student number on your solution and to staple the pages.

Exercise 1. Evaluate the following limits:

$$(a) \quad \lim_{z\to -i} \frac{z^4}{z^3+i} \qquad (b) \quad \lim_{z\to\infty} \frac{(z-2i)^2(7z-3)}{(1-iz)^2(1+5z)} \qquad (c) \quad \lim_{z\to i+1} \frac{z^5}{z^2-2i}$$

Exercise 2. For each of the following functions decide at which values of z the function is continuous and at which values it is not continuous. Give reasons, but detailed proofs are not expected.

(a) $f(z) = 5z^4 + 7z^3 + 3\overline{z}$,

(b) $f(z) = (z - \overline{z})/(iz)$. Check the cases (i) $z \neq 0$, and (ii) $z = 0$, separately.

Exercise 3. Let

$$f(z) = \left(\frac{\overline{z}}{z}\right)^n,$$

where $n \in \mathbb{N}$. By considering what happens when z approaches 0 along straight lines, show that $\lim_{z\to 0} f(z)$ does not exist.

Exercise* 4. (a) Write down the definition of the derivative of a complex function as a limit.

(b) Using the definition of the derivative from (a), find the derivative of

$$f(z) = 4z^2 + \frac{i}{z}$$

at $z = -i$,

(c) Using the definition of the derivative from (a), find the derivative of

$$f(z) = z^\alpha$$

at $z = z_0$ for any positive integer α. Using the binomial theorem will simplify your calculation. [The formula obtained is true for any real α, although a more general form of the binomial theorem must then be used, involving an infinite series.]

BJC 25/1/2013