MTH5103 Complex Variables 2012-2013

Coursework 4

Please put your solution to the *starred feedback exercise* in the red Complex Variables box in the basement by 3pm Friday 8 February. Remember to put your name (surname underlined) and student number on your solution and to staple the pages.

Exercise* 1.

(a) Let $f(z) = u(x, y) + iv(x, y)$, where as usual $z = x + iy$. Write down the Cauchy-Riemann equations.

(b) Each of the following functions can be written as $f(z) = u(x, y) + iv(x, y)$. Find the set of all points (x, y) at which u and v satisfy the Cauchy-Riemann equations:

$$(i) \qquad f(z) = z(xz + 1) + ixy(1 - 2x) \qquad (ii) \qquad f(z) = x^2(\frac{1}{2} + i) + 2x + i(z - 2xy)$$

(c) If u and v satisfy the Cauchy-Riemann equations at z_0, what extra conditions on u and v will ensure that $f = u + iv$ is differentiable at z_0?

Exercise 2. Consider the complex function

$$f(z) = ie^{-iz} .$$

(a) Express $f(z)$ in the form $u(x, y) + iv(x, y)$, give u and v.

(b) Use the Cauchy-Riemann equations and the existence and continuity of the partial derivatives of u and v to show that $f'(z)$ exists for all z.

(c) Using your results from b), find an expression for $f'(z)$ in terms of z.

Exercise 3.

(a) Prove the following result: An entire function $f : \mathbb{C} \to \mathbb{C}$ with $f(z) \in \mathbb{R}$ for all $z \in \mathbb{C}$ is necessarily a constant function. Using the Cauchy-Riemann equations will simplify the proof.

(b) Which of the following functions are entire? Give reasons.

(i) $f(z) = \sin |z|$,

(ii) $f(z) = z^5 + iz^7 + z^{12}$,

(iii) $f(z) = f(x + iy) = x$,

(iv) $f(z) = 2$.

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