Differential Equations (MTH4102)
Problem Sheet 1

NOTE: This coursework is intended as revision of differentiation, integration and curve sketching and is not for handing in for marking.

Problem 1

Compute the derivative $f'(x)$ of the following functions.

a) $f(x) = (x-1)(x^2+1)$ **b)** $f(x) = 1/(1-x^2)$ **c)** $f(x) = x/(1+x^2)$

d) $f(x) = (1-x^4)/(1+x^2)$ **e)** $f(x) = xe^{-x}$ **f)** $f(x) = x\sin(x)$

g) $f(x) = x\cos(x^2)$ **h)** $f(x) = x\ln|x|$ **i)** $f(x) = 1/(x\ln|x|)$

j) $f(x) = (\cos(x) + \sin(x)\tan(x))\cos(x)$

Problem 2

Compute the indefinite integral $\int f(x)\,dx$ of the following functions.

a) $f(x) = (x-1)(x^2+1)$ **b)** $f(x) = 1/(1-x^2)$ **c)** $f(x) = x/(1+x^2)$

d) $f(x) = (1-x^4)/(1+x^2)$ **e)** $f(x) = xe^{-x}$ **f)** $f(x) = x\sin(x)$

g) $f(x) = x\cos(x^2)$ **h)** $f(x) = x\ln|x|$ **i)** $f(x) = 1/(x\ln|x|)$

j) $f(x) = (\cos(x) + \sin(x)\tan(x))\cos(x)$

Problem 3

Sketch the graph of the following functions (it may help if you first compute the zeros, the maxima/minima, and the limit when x goes to infinity).

a) $f(x) = 2e^{-2x} - e^{-x}$

b) $f(x) = e^{-x}(\cos(x) + \sin(x))$

Problem A

a) Compute the derivative $f'(x)$ of the following functions

$$f(x) = e^{(e^x)}, \qquad f(x) = \left(e^e\right)^x, \qquad f(x) = \ln|\ln|x||\,.$$

b) Compute the indefinite integral $\int f(x)\,dx$ of the following function

$$f(x) = e^x\cos(x)\,.$$

c) Sketch the graph of the following function

$$f(x) = \frac{x}{1+x^2}\,.$$