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Differential Equations (MTH4102)
Problem Sheet 4

Problem 10

Determine the general solution of the following homogeneous linear differential equations. Fix for each solution the constant of integration according to the given initial condition.

a) $y' = -xy, \quad y(0) = -2$

b) $y' = x\cos(x)y, \quad y(0) = 1$

c) $y' = -y/(1+x), \quad y(0) = -1$

d) $y' = y/(4-x^2), \quad y(0) = 0$

e) $y' = y/(x^2+2x+2), \quad y(0) = 2$

Problem 11

Determine the general solution of the following inhomogeneous linear differential equations. Fix for each solution the constant of integration according to the given initial condition.

a) $y' = 3y + 5, \quad y(0) = -2$

b) $y' = -2xy + 2x, \quad y(0) = 1$

c) $y' = 3x^2y/(1+x^3) + x^2 + x^5, \quad y(0) = -1$

d) $y' = y + 2xe^{2x}, \quad y(0) = 3$

e) $y' = y\tan(x) + \cos(x), \quad y(0) = 2$

Problem 12

Consider the differential equation

$$y' = y - xy^3 \,.$$

a) Use the substitution $z(x) = 1/(y(x))^2$ to rewrite the differential equation in terms of the new dependent variable z.

b) Solve the linear inhomogeneous differential equation for z and determine the general solution $y(x)$ of the original differential equation.

For the homework please turn over

Problem D

a) Determine the general solution of the homogeneous linear differential equation

$$y' = \tan(x)y\,.$$

Fix the constant of integration according to the initial condition $y(\pi/4) = -\sqrt{8}$ and sketch the solution in a diagram in the interval $0 \leq x \leq \pi/2$.

b) Determine the general solution of the inhomogeneous linear differential equation

$$y' = \frac{xy}{1+x^2} + \sqrt{\frac{1+x^2}{1-x^2}}\,.$$

by the method of integrating factor.

c) Determine the general solution of the differential equation

$$y' = \frac{y}{2x} - (xy)^3\,.$$

Homework, and homework only, to be handed in during week 5 tutorials, Wed/Thurs 10/11 February.