General Solution to Linear, Inhomogeneous Differential Equation:

$$\frac{dy}{dx} = y' = a(x)y + b(x)$$

Method 1: Integrating Factor

Method 2: Variation of Parameters

Note: The notes for these two methods are on pages 11-13 of course notes.

Partial Derivatives

- Applies to a function $F = F(x, y)$ that is function of <u>two</u> independent variables x and y.

- Can take derivative with respect to either x or y (or both)

- Derivative given symbol 'curly dee' ∂ and called <u>partial derivative</u>

- Partial derivative with respect to x written as $\dfrac{\partial F}{\partial x}$

- Evaluate by taking derivative of F with respect to x <u>treating</u> y <u>as a fixed constant.</u>

Example : $F(x,y) = x^2 + xy^2$

$$\frac{\partial F}{\partial x} = 2x + y^2$$

Likewise, evaluate $\frac{\partial F}{\partial y}$ by differentiating with respect to y keeping x fixed.

$$\frac{\partial F}{\partial y} = 2xy$$

Second Derivatives

Write $\frac{\partial^2 F}{\partial y \partial x}$ to mean partial derivative of $\frac{\partial F}{\partial x}$ with respect to y :

$$\frac{\partial^2 F}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial F}{\partial x}\right) = \frac{\partial}{\partial y}(2x + y^2) = 2y$$

Likewise

$$\frac{\partial^2 F}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial F}{\partial y}\right) = \frac{\partial}{\partial x}(2xy) = 2y$$

Remark: Notice that $\underline{\underline{\dfrac{\partial^2 F}{\partial x \partial y} = \dfrac{\partial^2 F}{\partial y \partial x}}}$

This true in general (important!)

Chain Rule: If $y = y(x)$ then

$F(x, y(x))$ is explicit function of x.

Derivative with respect to x

$$F = x^2 + x\,(y(x))^2 \Rightarrow \frac{dF}{dx} = 2x + \frac{d}{dx}(xy^2)$$

$$\Rightarrow \frac{dF}{dx} = 2x + y^2 + x\frac{dy^2}{dx}$$

$$= 2x + y^2 + 2xy\frac{dy}{dx}$$

$$\underline{\underline{\frac{dF}{dx} = \frac{\partial F}{\partial x} + \frac{\partial F}{\partial y}\frac{dy}{dx}}}$$

This true in general (important)

Called <u>implicit</u> differentiation

Example:

$$F(x,y) = e^{x+y} + x^2 y = e^x . e^y + x^2 y$$

$$\frac{\partial F}{\partial x} = e^y . e^x + 2xy$$

$$\frac{\partial F}{\partial y} = e^x . e^y + x^2$$

$$\frac{\partial^2 F}{\partial y \partial x} = \frac{\partial}{\partial y}\left(e^y e^x + 2xy\right) = e^x e^y + 2x$$

$$\frac{\partial^2 F}{\partial x \partial y} = \frac{\partial}{\partial x}\left(e^x e^y + x^2\right) = e^x e^y + 2x$$

If $y = y(x)$

$$\frac{dF}{dx} = \underbrace{e^x e^y + 2xy}_{\partial F/\partial x} + \underbrace{\left(e^x e^y + x^2\right)}_{\partial F/\partial y} \frac{dy}{dx}$$