MTH5103 Complex Variables 2008-2009

Sample Test

Question A. [*25 marks*]

a) Let $z_1 = 2 - i$ and $z_2 = 3e^{-\frac{\pi i}{2}}$. Compute:
(i) $Im(z_2)$, (ii), $|z_1|$, and (iii) $|z_1 + z_2|$. [*15 marks*]

b) Describe the set of points $z \in \mathbb{C}$ satisfying:
(i) $|2iz - 1| = 4$, and (ii) $|z|^2 + 4Im(z) = -4$ [*10 marks*]

Question B. [*25 marks*]

a) Find all complex solutions of $\frac{i}{i+1} z^4 = -1$. [*15 marks*]

b) Show that under the map $z \mapsto w = z^2 + i$, the line $Im(z) = 1$ is mapped to the parabola given by $u = \left(\frac{v-1}{2}\right)^2 - 1$, where $w = u + iv$. [*15 marks*]

Question C. [*20 marks*]
Find the Möbius transformation $f(z) = (az + b)/(cz + d)$ which maps $0 \to 0$, $1 \to 2$, and $-i \to -2$. Check your results by substituting the values for z back in.

Question D. [*25 marks*]

a) Starting from the definition of the derivative of a complex function as a limit, find the derivative of $f(z) = iz^2$ for $z \in \mathbb{C}$. [*15 marks*]

b) Let $g(z) = g(x + iy) = xy - ix^2$. At what points z is the function g differentiable?
[*10 marks*]

KAM 04/02/2009