

MTH5120 Statistical Modelling I

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**Practical 1 Solutions** 

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# Minitab Project Report - Practical 1

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## Example 1



The relation between Y and X seems to follow an increasing pattern.

The linear model can be written as

$$Y_i = \beta_0 + \beta_1 x_i + \varepsilon_i \cdot \varepsilon_i \sim N(0, \sigma^2)$$
  
independently

and  $\beta_0$  and  $\beta_1$  are unknown model parameters: the intercept and the slope of the line.

#### Regression Analysis: y versus x

The regression $y = 0.061 + 1.$	equa 23 x	tion is			
Predictor	Coef	SE Coef			P
Constant U.	0612 2144	0.3916	) U.IK		8
X 1.2	3144	0.0401.	20.03	0.00	0
S = 1.04743	R-Sq	= 97.5%			
Analysis of Va	rianc	e			
Source	DF	SS	MS	F	P
Regression	1	781.28	781.28	712.12	0.000
Residual Error	18	19.75	1.10		
Total	19	801.03			

At x = 0 we would expect y = 0. Hence, in fact we should fit a no-intercept model; this is supported by a large p-value for the intercept.

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The regression equation is
y = 1.24 x
               Coef SE Coef
Predictor
                                  Т
                                         Ρ
Noconstant
            1.23722 0.02688 46.02
                                     0.000
х
Analysis of Variance
Source
                DF
                       SS
                               MS
                                          F
                                                Ρ
                   2204.2
                           2204.2 2117.86 0.000
Regression
                1
                     19.8
Residual Error 19
                              1.0
                20
                   2224.0
```

The interpretation of the model fit:

Total

When lactic acid concentration changes by one unit, then, on average, the measurement of the concentration changes by 1.24 of the unit.



At x = 7 the measured concentration is estimated to be  $1.24 \times 7 = 8.66$ .

The normal plot gives no clear reason to doubt the assumption of normality. As for the residuals versus fits, the spread is a bit different for different values of x but there is no systematic pattern (it gets larger, then smaller, then larger again). I would not think this is sufficient to cast doubt on the assumption of a constant variance. None of the residuals are very large. The plot against order is probably irrelevant as we don't know if the data was collected in this order.

The ANOVA test H<sub>0</sub>:  $\beta_1 = 0$  (the slope is zero) H<sub>1</sub>:  $\beta_1 \neq 0$  (the slope is significantly different from zero)

The value of T- statistic is 46.02 (F-statistic 2117.86) and the p-value is < 0.0001. Hence we reject the null hypothesis. The measured values depend linearly on the actual value of the concentration.

### Example 2





The data suggest a simple linear regression model

$$X_i = \beta_0 + \beta_1 X_i + \varepsilon_i$$
,  $\varepsilon_i \sim N(0, \sigma^2)$   
independently

### Regression Analysis: y versus x

The regression equation is y = 10.3 + 4.92 xPredictor Coef SE Coef Т Ρ 7.24 Constant 10.278 1.420 0.000 4.9193 0.3927 12.53 0.000 х Analysis of Variance DF SS MS F Ρ Source 841.77 1 841.77 156.89 0.000 Regression Residual Error 13 69.75 5.37 911.52 Total 14



The residual plots give us no reason to doubt the model assumptions.

$$H_0: \beta_1 = 0$$
$$H_1: \beta_1 \neq 0$$

The value of the F-statistic is 156.89 and the p-value is < 0.0001. Hence we reject the null hypothesis. The amount of damage caused by fire depends significantly on the distance of the property from the closest fire station.

y = 10.278 + 4.9193 \* 3.5 = 27.496

The estimated mean cost of fire damage for a residence 3.5 km from the nearest station is £27,496.

The intercept (£10,280) gives the average cost for a fire which happens very close to a fire station. Even if the fire engines didn't have to go very far there would be a large amount of damage. The slope (£4,919) is the average extra cost for each further kilometer that the fire engine has to travel.