1a) $f'(x) = (x^2+1) + 2x(x-1) = 3x^2 - 2x + 1$

b) $f'(x) = (-1)/(1-x^2)^2 \cdot (-2x) = 2x/(1-x^2)^2$

c) $f'(x) = 1/(1+x^2) - x \cdot 2x/(1+x^2)^2 = (1-x^2)/(1+x^2)^2$

d) $f(x) = (1-x^2)(1+x^2)/(1+x^2) = (1-x^2)$

$\Rightarrow f'(x) = -2x$

e) $f'(x) = e^{-x} - x \cdot e^{-x} = (1-x)e^{-x}$

f) $f'(x) = \sin x + x \cos x$

g) $f'(x) = \cos(x^2) - x \cdot 2x \sin(x^2) = \cos(x^2) - 2x^2 \sin(x^2)$

h) $f'(x) = \ln|x| + x \cdot 1/x = \ln|x| + 1$

i) $f'(x) = (-1)/(x\ln|x|)^2 \cdot (\ln|x| + 1) = \dfrac{-\ln|x| - 1}{(x\ln|x|)^2}$

j) $f(x) = \cos^2 x + \sin x \cdot \dfrac{\sin x}{\cos x} \cdot \cos x = 1 \Rightarrow f'(x) = 0$

2a) $\int f(x)dx = \int x^3 - x^2 + x - 1 = \frac{1}{4}x^4 - \frac{1}{3}x^3 + \frac{1}{2}x^2 - x + C$

b) $\dfrac{1}{1-x^2} = \dfrac{1}{2}\left(\dfrac{1}{1-x} + \dfrac{1}{1+x}\right)$

$\Rightarrow \int f(x)dx = \frac{1}{2}(-\ln|1-x| + \ln|1+x|) + C$

c) $\displaystyle\int \frac{x}{1+x^2}dx \overset{1+x^2=u,\ du=2x\,dx}{=} \int \frac{1/2\,du}{u} = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|1+x^2| + C$

d) $\int f(x)dx \underset{\text{cf. 1d)}}{=} \int 1 - x^2 dx = x - \frac{1}{3}x^3 + C$

e) $\int \overset{u}{x}\,\overset{v'}{e^{-x}}\,dx = \overset{u}{x}(\overset{v}{-e^{-x}}) - \int \overset{u'}{1}(\overset{v}{-e^{-x}})dx$

$= -xe^{-x} + \int e^{-x}dx = -xe^{-x} - e^{-x} + C_1$

f) $\int x\sin(x)dx = x(-\cos(x)) - \int 1\cdot(-\cos(x))dx$

$= -x\cos(x) + \sin(x) + C_1$

g) $\int x\cos(x^2)dx \underset{x^2=u,\ du=2xdx}{=} \int \frac{1}{2}\cos(u)du = \frac{1}{2}\sin(u) + C_1 = \frac{1}{2}\sin(x^2) + C_1$

h) $\int \underset{v'}{x}\,\underset{u}{\ln|x|}\,dx = \underset{v}{\frac{x^2}{2}}\,\underset{u}{\ln|x|} - \int \underset{v}{\frac{x^2}{2}}\cdot\underset{u'}{\frac{1}{x}}\,dx = \frac{x^2}{2}\ln|x| - \frac{x^2}{4} + C_1$

i) $\int \frac{1}{x\ln|x|}dx \underset{\ln|x|=u,\ du=\frac{1}{x}dx}{=} \int \frac{1}{u}du = \ln|u| + C_1 = \ln|\ln|x|| + C_1$

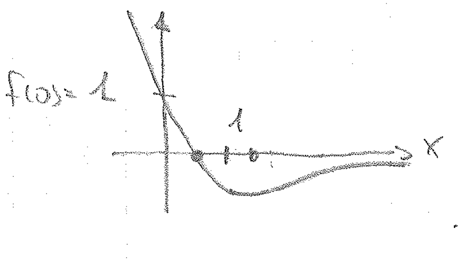
j) $\int f(x)dx \underset{(cf.\ 1j))}{=} \int 1\,dx = x + C_1$

3a) zero: $0 \overset{!}{=} f(x) = e^{-x}(2e^{-x}-1) \Rightarrow e^{x} = 2 \Rightarrow x = \ln 2 = 0.69...$

maximal / minima: $0 \overset{!}{=} f'(x) = -4e^{-2x} + e^{-x} = e^{-x}(-4e^{-x}+1)$

$\Rightarrow e^{x} = 4 \Rightarrow x = \ln 4 = 1.386...$

Limit $x\to\infty$: $\lim_{x\to\infty} f(x) = 0$ since $e^{-x}, e^{-2x} \to 0$



b) zeros: $0 \overset{!}{=} f(x) = e^{-x}(\cos x + \sin x)$

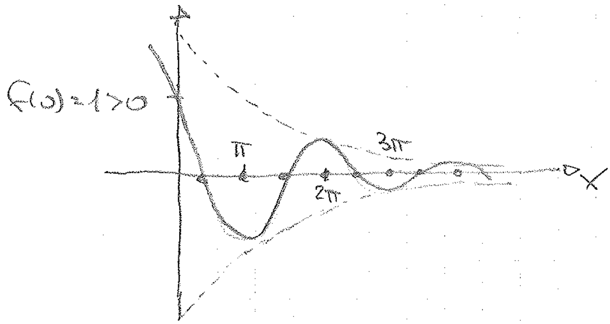
$\Rightarrow \cos x = -\sin x \Rightarrow \tan x = -1 \Rightarrow x = -\frac{\pi}{4} + n\pi$ (i.e. $-\frac{\pi}{4}, -\frac{\pi}{4}+\pi, \dots$)

maximal / minimal: $f'(x) = -e^{-x}(\cos x + \sin x) + e^{-x}(-\sin x + \cos x)$

$= -2e^{-x}\sin x$

$f'(x) \overset{!}{=} 0 \Rightarrow x = n\pi$ (i.e. $0, \pm\pi, \pm 2\pi, \dots$)

Limit: $\lim\limits_{x\to\infty} f(x) = 0$ since $e^{-x} \to 0$ and $|\sin x| \le 1$, $|\cos x| \le 1$



1 a) $f(x) = e^{(e^x)} \Rightarrow f'(x) = e^x \cdot e^{(e^x)} \;(= e^{x+e^x})$ 10P

$f(x) = (e^e)^x = e^{x\cdot e} \Rightarrow f'(x) = e \cdot e^{xe} \;(= e^{1+xe})$ 10P

$f(x) = \ln|\ln|x|| \Rightarrow f'(x) = \frac{1}{\ln|x|} \cdot \frac{1}{x} \;\left(= \frac{1}{x\ln|x|}\right)$ 10P

b) $I := \int e^x \cos x \, dx = e^x \cos x - \int e^x(-\sin x)\,dx$

$= e^x \cos x + \int e^x \sin x \, dx$ 10P

$= e^x \cos x + e^x \sin x - \int e^x \cos x \, dx$

$= e^x \cos x + e^x \sin x - I \quad (+C_1)$ 10P

$\Rightarrow 2I = e^x \cos x + e^x \sin x \quad (+C_1)$

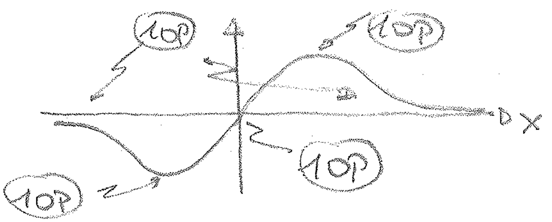
$\Rightarrow I = \frac{1}{2} e^x (\cos x + \sin x) + C$ 10P

c) zero: $0 = f(x) = \frac{x}{1+x^2} \Rightarrow x = 0$

maxima minima $0 = f'(x) = \frac{1-x^2}{(1+x^2)^2} \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$

(cf 1c)

Limiti $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{1/x}{1/x^2 + 1} = \frac{0}{0+1} = 0$



$(f'(0) = 1 > 0)$