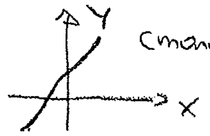
C/w 2.

4a) $y=\int 3x^2+2x+3\,dx = x^3+x^2+3x+C_1$

$y(0)=1=0+C_1 \Rightarrow C=1.$

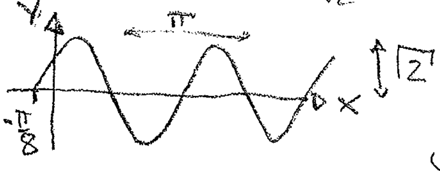


(monotone) $y'>0$ since $3x^2+2x+3=(x+1)^2+2x^2+2>0.$

b) $y=\int -2\sin(2x)+2\cos(2x)\,dx = \cos(2x)+\sin(2x)+C_1$

$y(0)=1=1+0+C_1 \Rightarrow C=0$

$y=\sqrt{2}\left\{\frac{1}{\sqrt{2}}\sin(2x)+\frac{1}{\sqrt{2}}\cos(2x)\right\}=\sqrt{2}\sin\left(2x+\frac{\pi}{4}\right)$



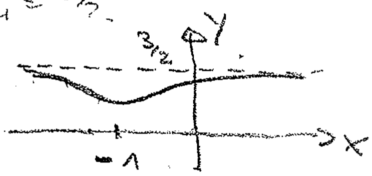
$u=\frac{x^2}{2}+x,\ du=(x+1)dx$

c) $y=\int (x+1)e^{-2(\frac{x^2}{2}+x)}dx = \int e^{-2u}du$

$=-\frac{1}{2}e^{-2u}+C_1=-\frac{1}{2}e^{-x^2-2x}+C_1$

$y(0)=1=-\frac{1}{2}e^0+C_1 \Rightarrow C_1=3/2.$

$y=-\frac{1}{2}e\,e^{-(x+1)^2}+3/2$



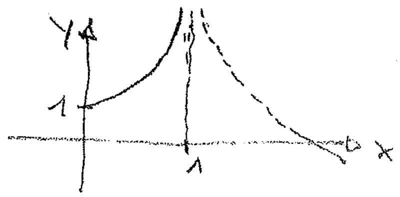
d) $y=\int e^{-x}\cos(x)\,dx = \frac{1}{2}e^{-x}(-\cos(x)+\sin(x))+C_1$

cf. 4b)

$y(0)=1=\frac{1}{2}(-1+0)+C_1 \Rightarrow C=3/2$

e) $y = \int \frac{3}{1-x}\,dx = -3\ln|1-x| + C'$

$y(0) = 1 = -3\cdot 0 + C' \Rightarrow C = 1$



5.) I) $y' = 2x = 2\sqrt{(1+x^2)-1} = 2\sqrt{y-1} \Rightarrow$ D)

II) $y' = \frac{1}{1-x} + \frac{x}{(1-x)^2} = \frac{y}{x} + \frac{y^2}{x} = \frac{y(1+y)}{x} \Rightarrow$ C)

III) $y' = \frac{1}{2}\frac{2x}{\sqrt{1+x^2}} = \frac{x}{y-1} \Rightarrow$ A)

IV) $y' = e^x + xe^x = (1+x)e^x = (1+x)\frac{y}{x} \Rightarrow$ B)

V) $y' = -\sin(x-1) = -\sqrt{1-y^2} \Rightarrow$ E)

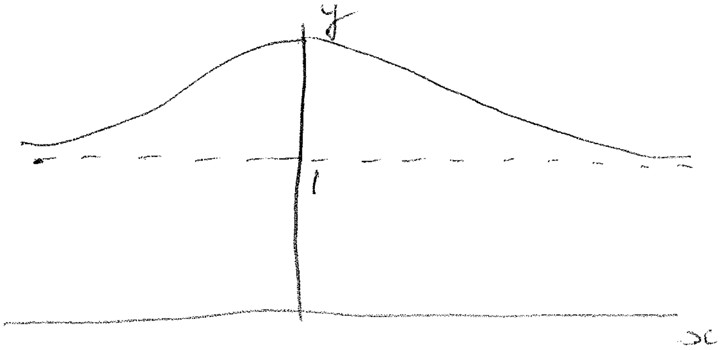
(in some domain)

3a) $y = -\int \frac{4x\,dx}{(1+2x^2)^2}$ $\quad u = 1+2x^2 \Rightarrow du = 4x\,dx$

$$\Rightarrow y = -\int \frac{du}{u^2} \Rightarrow y = \frac{1}{u} + C = \frac{1}{1+2x^2} + C$$

$$y(0) = 2 \Rightarrow 2 = 1 + C \Rightarrow C = 1$$

$$y = \frac{2+2x^2}{1+2x^2}$$



3b) $y = x^2(1-x)^{-1/2} \Rightarrow y' = \frac{2x}{(1-x)^{1/2}} + \frac{x^2}{2(1-x)^{3/2}}$

$$\Rightarrow y' = \frac{2y}{x} + \frac{y^3}{2x^4}$$ NO!

3c) $y = \int 3^x\,dx = \int e^{x\ln 3}\,dx = \frac{1}{\ln 3} 3^x + C.$