7a) $y\,dy = -x\,dx \Rightarrow \int y\,dy = -\int x\,dx \Rightarrow \frac{1}{2}y^2 = -\frac{1}{2}x^2 + C_1$

$\Rightarrow y = \pm\sqrt{2C_1 - x^2}$

$y(0) = -2 = -\sqrt{2C_1} \Rightarrow C_1 = 2$

!!

b) $\frac{y}{y^2+1}dy = dx \Rightarrow \int \frac{y}{y^2+1}dy = \int dx \Rightarrow \frac{1}{2}\ln(1+y^2) = x + C_1$

$\Rightarrow 1+y^2 = e^{2C}e^{2x} = De^{2x} \Rightarrow y = \pm\sqrt{De^{2x}-1}$

$y(0) = 1 = +\sqrt{D-1} \Rightarrow D = 2$

c) $\frac{1}{1+y^2}dy = e^x dx \Rightarrow \int \frac{dy}{1+y^2} = \int e^x dx \Rightarrow \arctan(y) = e^x + C_1$

$\Rightarrow y = \tan(e^x + C_1)$

$y(0) = -1 \Rightarrow \arctan(-1) = e^0 + C_1 \Rightarrow C_1 = -\frac{\pi}{4} - 1$

(not unique)

d) $y' = (e^x+1)(y-2) \Rightarrow \frac{dy}{y-2} = (e^x+1)\,dx$

$\Rightarrow \int \frac{dy}{y-2} = \int (e^x+1)\,dx \Rightarrow \ln|y-2| = e^x + x + C_1$

$\Rightarrow |y-2| = e^c e^{(e^x+x)} \Rightarrow y - 2 = De^{(e^x+x)}$

$\Rightarrow y = 2 + De^{(e^x+x)}$

$y(0) = 0 = 2 + De^{1+0} \Rightarrow D = -2e^{-1}$

e) $\frac{dy}{y} = \frac{x}{x-1}dx \Rightarrow \int \frac{dy}{y} = \int \left(1 + \frac{1}{x-1}\right)dx$

$\Rightarrow \ln|y| = x + \ln|x-1| + C_1 \Rightarrow |y| = e^{C}\cdot|x-1|\cdot e^{x}$

$\Rightarrow y = D(x-1)e^{x}$

$y(0) = 2 = D(-1)e^{0} \Rightarrow D = -2$

8a) $y = x\cdot z \Rightarrow y' = xz' + z$

$\Rightarrow xz' + z = y' = -\frac{y}{x} - \frac{x}{y} = -\frac{xz}{x} - \frac{x}{xz} = -z - \frac{1}{z}$

$\Rightarrow xz' = -2z - 1/z \Rightarrow \quad z' = -\frac{1}{x}\,\frac{2z^2+1}{z}$

b) $\frac{z}{2z^2+1}dz = -\frac{1}{x}dx$

$\Rightarrow \frac{1}{4}\ln(2z^2+1) = -\ln|x| + C_1$

$\Rightarrow \ln(2z^2+1) = \ln|x^{-4}| + 4C_1$

$\Rightarrow 2z^2+1 = Dx^{-4} \Rightarrow z^2 = \frac{1}{2}(Dx^{-4}-1)$

$\Rightarrow y^2 = x^2z^2 = \frac{D-x^4}{2x^2} \Rightarrow y = \pm\frac{\sqrt{D-x^4}}{\sqrt{2}\cdot x}$

9) $y = xz \Rightarrow y' = xz' + z$

$\Rightarrow xz' + z = \frac{x^3+(xz)^3}{x(xz)^2} = \frac{1+z^3}{z^2} \Rightarrow xz' = \frac{1}{z^2}$

$$\Rightarrow z^2 dz = \frac{1}{x}dx \Rightarrow \frac{1}{3}z^3 = \ln|x| + C'$$

$$\Rightarrow z = \sqrt[3]{3\ln|x| + 3C'}$$

9. $$y' = x^4(1-y^4) \Rightarrow \frac{dy}{1-y^4} = x^4\,dx$$

$$\frac{1}{1-y^4} = \frac{1}{(1-y)(1+y)(1+y^2)} = \frac{1}{4(1-y)} + \frac{1}{4(1+y)} + \frac{1}{2(1+y^2)}$$

$$\Rightarrow \int \frac{dy}{1-y^4} = -\frac{1}{4}\ln|1-y| + \frac{1}{4}\ln|1+y| + \frac{1}{2}\arctan(y)$$

$$\Rightarrow \frac{1}{4}\ln\left|\frac{1+y}{1-y}\right| + \frac{1}{2}\arctan(y) = \frac{1}{5}x^5 + C.$$

c a) $y' = \dfrac{x^2}{2y+1} \Rightarrow \int (2y+1)\,dy = \int x^2\,dx$

$\Rightarrow y^2 + y = \frac{1}{3}x^3 + C$ (general solution, ~~explicit~~ implicit form)

$\Rightarrow \left(y+\frac{1}{2}\right)^2 = \frac{1}{3}x^3 + C + \frac{1}{4}$

$\Rightarrow y + \frac{1}{2} = \pm\left(\frac{1}{3}x^3 + C + \frac{1}{4}\right)^{1/2}$

$y = -\frac{1}{2} \pm \sqrt{\frac{x^3}{3} + C + 1/4}$ $\boxed{15}$

$\boxed{10}$

$y(0) = -1 \Rightarrow y(0) = -1 = -\frac{1}{2} - \sqrt{C + 1/4} \Rightarrow C = 0$ $\boxed{5}$

$\Rightarrow \underline{y(x) = -\frac{1}{2} - \sqrt{\frac{x^3}{3} + \frac{1}{4}}}$

b) $y' = \dfrac{y}{x^2+4x+5} \Rightarrow \int \dfrac{dy}{y} = \int \dfrac{dx}{(x+2)^2+1}$

$\Rightarrow \ln|y| = \arctan(x+2) + C$ $\boxed{10}$

$\Rightarrow |y| = e^C e^{\arctan(x+2)}$

$\Rightarrow y = D e^{\arctan(x+2)}$ $\boxed{10}$ $(D = \pm e^C)$

$y(-1) = 1 \Rightarrow 1 = D e^{\arctan(1)} \Rightarrow D = e^{-\pi/4}$

$\Rightarrow y(x) = e^{-\pi/4} e^{\arctan(x+2)}$ $\boxed{10}$

(c) $\quad y' = \dfrac{2x^2+y^2}{xy} = 2\dfrac{x}{y} + \dfrac{y}{x}$

$$y(x) = xz(x) \Rightarrow \frac{dy}{dx} = z + x\frac{dz}{dx}$$

$$\Rightarrow z + x\frac{dz}{dx} = \frac{2}{z} + z \Rightarrow \frac{dz}{dx} = \frac{2}{xz} \quad \boxed{20}$$

$$\cancel{\Rightarrow 2\int dz.z} \Rightarrow \int dz.z = 2\int\frac{dx}{x}$$

$$\Rightarrow \frac{1}{2}z^2 = 2\ln|x| + C \quad \boxed{10}$$

$$\Rightarrow \frac{y^2}{x^2} = 4\ln|x| + 2C$$

$$\Rightarrow y(x) = \pm x\sqrt{4\ln|x| + 2C} \quad \boxed{10}$$