C/W 4.

10a) $y = De^{\int -x\,dx} = De^{-1/2 x^2}$

$y(0) = -2 = De^{0} \Rightarrow D = -2$

b) $y = De^{\int x\cos x\,dx} = De^{x\sin x + \cos x}$

$y(0) = 1 = De^{1} \Rightarrow D = e^{-1}$ (prob. redef.)

c) $y = De^{\int \frac{-1}{1+x}dx} = De^{-\ln|1+x|} \overset{!}{=} \frac{D}{1+x}$

$y(0) = -1 = \frac{D}{1+0} \Rightarrow D = -1$

d) $y = De^{\int \frac{dx}{4-x^2}} = De^{\int \left(\frac{1}{4}\frac{1}{2-x} + \frac{1}{4}\frac{1}{2+x}\right)dx}$

$= De^{-1/4 \ln|2-x| + \frac{1}{4}\ln|2+x|} = D\left(\frac{2+x}{2-x}\right)^{1/4}$

$y(0) = 0 = D \cdot 1 \Rightarrow D = 0$

e) $y = De^{\int \frac{dx}{x^2+2x+2}} = De^{\int \frac{1}{(x+1)^2+1}dx}$

$= De^{\arctan(x+1)}$

$y(0) = 2 = De^{\arctan(1)} = De^{\pi/4} \Rightarrow D = 2e^{-\pi/4}$

11a) Integrating factor $u(x) = e^{-\int 3dx} = e^{-3x}$

$\frac{de^{-3x}y}{dx} = 5e^{-3x} \Rightarrow e^{-3x}y = \int 5e^{-3x}dx = -\frac{5}{3}e^{-3x} + C$

$\Rightarrow y = -\frac{5}{3} + C_1' e^{+3x} \qquad y(0) = -2 = -\frac{5}{3} + C_1' \Rightarrow C = -\frac{1}{3}$

b) $u(x) = e^{-\int -2x\,dx} = e^{x^2}$

$\Rightarrow \frac{d\, e^{x^2} y}{dx} = 2xe^{x^2} \Rightarrow e^{x^2} y = \int 2xe^{x^2} dx = e^{x^2} + C_1'$

$\Rightarrow y = 1 + C_1' e^{-x^2} \qquad y(0) = 1 = 1 + C_1' \Rightarrow C_1' = 0$

c) $u(x) = e^{-\int \frac{3x^2}{1+x^3} dx} = e^{-\ln|1+x^3|} = \frac{1}{1+x^3}$

$\Rightarrow \frac{d \frac{1}{1+x^3} y}{dx} = \frac{1}{1+x^3}(x^2 + x^5) \Rightarrow \frac{1}{1+x^3} y = \int x^2 dx = \frac{1}{3}x^3 + C_1'$

$\Rightarrow y = \frac{1}{3}x^3(1+x^3) + C_1'(1+x^3)$

$y(0) = -1 = \frac{1}{3} \cdot 0 + C_1' \Rightarrow C = -1$

d) $u(x) = e^{-\int dx} = e^{-x}$

$\Rightarrow \frac{d\, e^{-x} y}{dx} = 2xe^{2x}e^{-x} \Rightarrow e^{-x} y = \int 2xe^{x} dx = 2xe^{x} - 2e^{x} + C_1'$

$\Rightarrow y = (2x-2)e^{2x} + C_1' e^{x}$

$y(0) = 3 = -2 + C_1' \Rightarrow C_1' = 5$ (in some domain)

e) $u(x) = e^{-\int \tan x} = e^{\ln \cos x} = \cos x$

$\frac{d}{dx}(y \cos x) = \cos^2(x) \Rightarrow y\cos x = \int \cos^2(x) = \frac{1}{2}(x + \cos x \sin x)$ $+C$

$y(0) = 2 \Rightarrow \quad C = 2 \Rightarrow y = \frac{1}{2}\sec x\,(x + \cos x \sin x) + C$

12a) $z = \frac{1}{y^2} \Rightarrow z' = -\frac{2}{y^3} \cdot y'$

$\Rightarrow z' = -\frac{2}{y^3} y' = -\frac{2}{y^3}(y - x y^3) = -\frac{2}{y^2} + 2x = -2z + 2x$

$\Rightarrow z' = -2z + 2x$

b) Integrating factor $u(x) = e^{-\int -2\,dx} = e^{2x}$

$\Rightarrow \frac{d\, e^{2x} z}{dx} = 2x e^{2x} \Rightarrow e^{2x} z = \int 2x e^{2x}\,dx = x e^{2x} - \frac{1}{2} e^{2x} + C_1$

$\Rightarrow z = x - \frac{1}{2} + C_1 e^{-2x}$

$y = \pm \frac{1}{\sqrt{z}} = \frac{\pm 1}{\sqrt{x - 1/2 + C_1 e^{-2x}}}$

c/w 4.

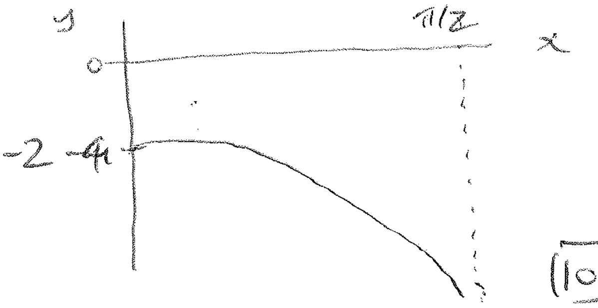
Q a) ~~$y' = \tan x$~~

$$\frac{dy}{dx} = \tan(x)\,y \quad \Rightarrow \quad \int \frac{dy}{y} = \int \tan(x)\,dx = \int \frac{\sin(x)}{\cos(x)}\,dx$$

$$\Rightarrow \ln|y| = -\ln|\cos(x)| + C \quad \Rightarrow \quad |y| = e^{C}|\sec(x)|$$

$$\Rightarrow \quad y = \pm e^{C}\sec(x) \quad \Rightarrow \quad y = D\sec(x) \qquad \boxed{10}$$

$$y\left(\frac{\pi}{4}\right) = -\sqrt{8} \quad \Rightarrow \quad -\sqrt{8} = D\sec\left(\frac{\pi}{4}\right) = D\sqrt{2} \Rightarrow D = -2 \quad \boxed{10}$$



$\boxed{10}$

Q b) $$y' = \frac{xy}{1+x^2} + \sqrt{\frac{1+x^2}{1-x^2}}$$

Integrating Factor $$u = e^{-\int \frac{x\,dx}{1+x^2}} = e^{-\frac{1}{2}\ln(1+x^2)} = (1+x^2)^{-1/2} \qquad \boxed{10}$$

$$\frac{d}{dx}\left(\frac{y}{\sqrt{1+x^2}}\right) = \frac{1}{\sqrt{1+x^2}}\sqrt{\frac{1+x^2}{1-x^2}} = \frac{1}{\sqrt{1-x^2}} \qquad \boxed{10}$$

$$\Rightarrow \quad \frac{y}{\sqrt{1+x^2}} = \int \frac{dx}{\sqrt{1-x^2}} = \arcsin(x) + C \qquad \boxed{10}$$

$$y(x) = \sqrt{1+x^2}\,\left(C + \arcsin(x)\right) \qquad \boxed{10}$$

Dc) Use $z = \frac{1}{y^2} \Rightarrow dz = \frac{-2}{y^3}\,dy \Rightarrow z' = \frac{-2}{y^3}\,y' \Rightarrow z' = -\frac{2}{y^3}\left(\frac{y}{2x} - x^3 y^3\right)$ $\boxed{10}$

$$\Rightarrow z' = -\frac{1}{xy^2} + 2x^3 \Rightarrow z' = -\frac{z}{x} + 2x^3 \quad \boxed{10}$$

integrating Factor $u = e^{\int dx/x} = e^{\ln x} = x$

$$\Rightarrow \frac{d}{dx}(zx) = 2x^4$$

$$\Rightarrow zx = \int 2x^4\,dx = \frac{2}{5}x^5 + c$$

$$\Rightarrow \frac{1}{y^2} = \frac{2}{5}x^4 + \frac{c}{x}$$

$$y = \pm\left(\frac{2}{5}x^4 + \frac{c}{x}\right)^{-1/2} \quad \boxed{10}$$