
School of Mathematical Sciences
Mile End, London E1 4NS · UK

Examiner: Dr T Prellberg

MTH5105 Differential and Integral Analysis
MID-TERM TEST

Date: 20-02-2009 Time: 2:10–2:50

Complete the following information:

| | |
|--|--|
| Name | |
| Student Number (9 digit code) | |

The test has THREE questions. You should attempt ALL questions. Write your calculations and answers in the space provided. Cross out any work you do not wish to be marked.

| Question | Marks |
|--------------------|--------------|
| 1 | |
| 2 | |
| 3 | |
| Total Marks | |

Nothing on this page will be marked!

Nothing on this page will be marked!

Question 1.

Let $f(x) = 1/x$.

- (a) Determine the Taylor polynomials $T_{3,1}$ and $T_{4,1}$ of degree 3 and 4 at $a = 1$ for f .

[15 marks]

- (b) Using the Lagrange form of the remainder, or otherwise, show that

$$T_{3,1}(x) < f(x) < T_{4,1}(x) \quad \text{for all } x > 1 .$$

[15 marks]

Answer 1.

Answer 1. (*Continue*)

Question 2.

(a) Give the definition of $f : \mathbb{R} \rightarrow \mathbb{R}$ being differentiable at a point $a \in \mathbb{R}$.

[10 marks]

(b) Using the definition, determine whether or not

$$f(x) = \begin{cases} \frac{x}{1 + \exp(1/x)} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

is differentiable at $x = 0$. (For this you may wish to consider the left and right derivatives of $f(x)$ at $x = 0$.) Find $f'(0)$, if it exists.

[20 marks]

Answer 2.

Answer 2. (*Continue*)

Question 3.

(a) State the Mean Value Theorem.

[15 marks]

(b) Show that for all $x, y \in \mathbb{R}$

$$|\sin(y) - \sin(x)| \leq |y - x| .$$

[25 marks]

You may assume standard properties of trigonometric functions.

Answer 3.

Answer 3. (*Continue*)