MTH5103 COMPLEX VARIABLES
MID-TERM TEST

Date: 22 February 2013 *Time:* 12.10–12.50

Complete the following information:

Name	
Student Number (9 digit code)	

The test has FOUR questions. You should attempt ALL questions. Write your calculations and answers in the space provided. Cross out any work you do not wish marked. Calculators are NOT allowed.

Question	Marks
A	
B	
C	
D	
Total Marks	

Question A. [*20 marks*]
Let $z_1 = -2 + 3i$ and $z_2 = 4e^{i\pi/3}$. Compute:

(a) $\mathrm{Im}(z_2)$, [*4 marks*]

(b) $|z_1|$, [*4 marks*]

(c) $|z_1 - z_2|$, [*6 marks*]

(d) ${z_1}^{-1}$. [*6 marks*]

There is no need to evaluate radicals. Present your result in the standard form $z = x + iy$ in (d).

Answer A.

Answer A. (*Continued*)

Question B. [*32 marks*]

(a) Find all complex solutions of $z^3 = 8i$, giving the results in polar form and indicating their location in the Argand diagram. [*16 marks*]

(b) Derive the curve corresponding to

$$\left|\frac{z-1}{z+1}\right| = 2$$

and describe this geometrically. [*16 marks*]

Answer B.

Answer B. (*Continued*)

Question C. [*22 marks*]

(a) Evaluate the limit

$$\lim_{z\to i} \frac{(2z+7)(z+i)}{(z-2i)^3}\,.$$

[*10 marks*]

(b) Under the transformation

$$z \mapsto w = z^2 + 2z - 5\,,$$

where $z = x + iy$ and $w = u + iv$, determine which points in the z-plane are mapped to $v = 2$ in the w-plane and describe these geometrically.

[*12 marks*]

Answer C.

Answer C. (*Continued*)

Question D. [*26 marks*]

(a) Let $f(z) = u + iv$, where $z = x + iy$. Write down the *Cauchy-Riemann* equations relating u and v. What other conditions are required for $f(z)$ to be differentiable at z? [*8 marks*]

(b) Use the *Cauchy-Riemann* equations to find the values of z for which each of the following functions is differentiable:

$$f(z) = 3xy - ix^3, \quad f(z) = |z|^2, \quad e^z .$$

[*18 marks*]

Answer D.

Answer D. (*Continued*)